SWAMI VIVEKANANDA SCHOOL OF ENGINEERING & TECHNOLOGY

LECTURE NOTE

FLUID MECHANICS

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FLUID MECHANICS

Properties of fluid :

Density or mass density :

It is defined as the mass per unit volume of a liqued at a standrad temperature and pressure. It is usually denoted as ρ. Unit is kg/m^3.

Mathematically, $\rho = \frac{m}{V}$ [m, V = mass & volume of liqued]

Weight density or specific weight :

It is defined as the weight per unit volume of a liqued at a standrad temperature and pressure. It is usually denoted by W or γ. Unit is KN/m^3 or N/m^3.

Mathematically, $W = \rho \cdot g$.

Note : For water, $W = 9.81\ KN/m^3 = 9.81 \times 10^3\ N/m^3$.

Specific volume :

It is defined as the volume per unit mass of the liqued. It is denoted by v.

Mathematically, $v = \frac{V}{m} = \frac{1}{\rho}$

Specific gravity :

It is defined as the ratio of the specific weight or density of a liqued to the specific weight of pure water at a standrad temperature (4°c). It has no units. For example, the sp. gravity of mercury is 13.6, hence ρ of mercury $= 13.6 \times 1000 = 13600\ kg/m^3$.

Viscosity :

It is also known as absolute or dynamic viscosity. It is defined as the property of a liqued which offers resistance to the movement of one layer of liqued over another adjacent layer of liqued.

It is due to cohesion and interaction between Particles. $\tau = \mu \frac{du}{dy}$ [μ = co efficient of dynamic viscosity, $\frac{du}{dy}$ = The rate of shear strain or velocity gradient]

Kinematic viscosity:

It is defined as the ratio of dynamic viscosity to the density of liquid.

Surface tension:

It is that property of a liquid which enables it to resist tensile stress. It is denoted by σ (sigma). It's unit is N/m.

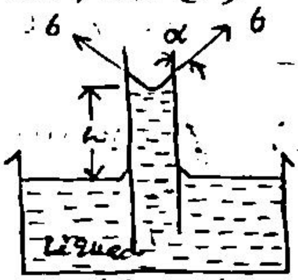
capillary Phenomenon:

It is defined as a phenomenon of rise or fall of a liquid surface in a small vertical tube held in a liquid relative to general level of the liqued. The height of rise or fall (h) in the tube is given by,

For rise, $\alpha = 0$

$$\boxed{h = \frac{4\sigma}{wd}}$$

$$\boxed{h = \frac{4\sigma \cos\alpha}{wd}} \quad [w = \rho g]$$



capillary rise

where, σ = Surface tension,

α = Angle of contact of the liquid surface.

w = Specific weight of liqued, and

d = Diameter of the capillary tube.

(II) Fluid pressure and its measurements:

Pressure of a liqued:

When a liqued is contained in a vessel, it exerts force at all Points on the sides and bottom of the vessel. The force per unit area is called intensity of pressure.

Mathematically, $p = \frac{P}{A}$

P = Force acting on the liqued, and

A = Area on which the force acts.

The intensity of Pressure at any Point, in a liquid at rest is equal to the Product of weight density of the liquid (w) and the vertical height from the free surface of the liquid (h).

Mathematically, $P = w \cdot h$ (h = Pressure head)

From this expression, the intensity of Pressure at any Point, in a liquid, is directly proportional to depth of liquid from the surface.

Unit of Pressure is Pascal.

$1\ Pa = 1\ N/m^2$ and $1\ MPa = 1\ MN/m^2 = 1\ N/mm^2$

Atmospheric Pressure, Gauge Pressure and Absolute Pressure:

The atmospheric air exerts a normal Pressure upon all surfaces with which it is in contact and it is known as atmospheric Pressure. It is also known as barometric Pressure. The atmospheric Pressure at sea level (above absolute zero) is called standard atmospheric Pressure and its value is given as follows:

Standard atmospheric Pressure $= 101.3\ kN/m^2$ or kPa
$= 10.3$ m of water.
$= 760$ mm of Hg.

The Pressure measured with the help of a Pressure gauge is known as gauge Pressure, in which atmospheric Pressure is taken as datum. All the Pressure gauges record the difference between the actual and atmospheric Pressure. The actual Pressure is known as absolute Pressure.

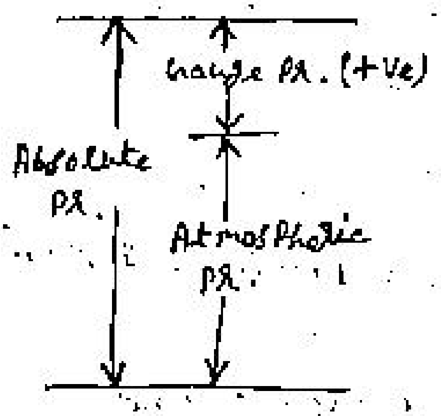
Mathematically,

Absolute Pressure = Atmospheric Pr + Gauge Pr (Positive)

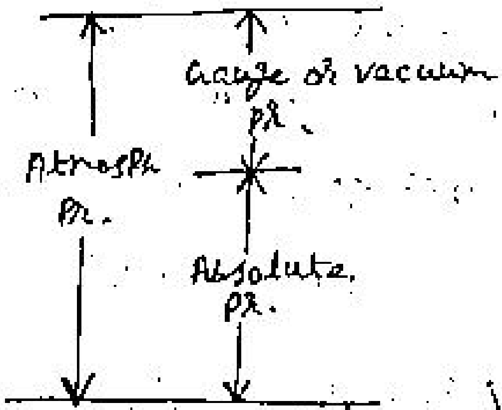
Vacuum Pressure :

For pressures below atmospheric, the gauge pressure will be negative. This negative gauge pressure is called as vacuum pressure.

Absolute pressure = Atmospheric Pr − Negative gauge or vacuum Pr.



(a) Relation betwn absolute atm. and gauge pressure.



(b) Relation betwn absolute, atm and vacuum Pr.

III Hydrostatics :

Hydrostatic Pressure :

Hydraulic Pr. deals with the fluids (liquids and gases) at rest. This means that there will be no relative motion betwn adjacent fluid layers.

$\frac{du}{dy} = 0 \quad \therefore \tau = 0.$

Total Pressure :

The total pressure is defined as the force exerted by a static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always normal to the surface.

Centre of Pressure :

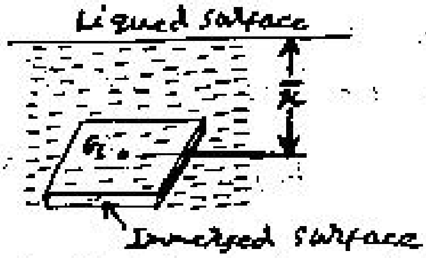
The centre of pressure is defined as the point of application of the resultant pressure on the surface.

The total pressure and centre of pressure on the immersed surfaces are as follows :

(1) Horizontally immersed surface :

The total pressure on a horizontally immersed surface, as shown in figure.

$$P = w \cdot A \cdot \bar{x}$$



Where, w = Specific weight of the liquid.

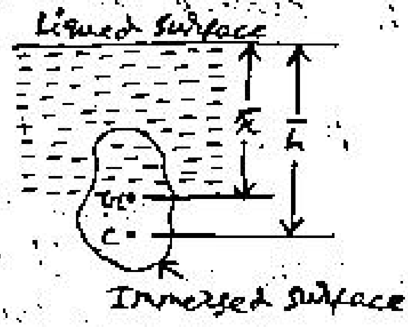
A = Area of the immersed surface and

$\bar{x}$ = Depth of the c.g of the immersed surface from the liquid surface.

(2) Vertically immersed surface :

The total pressure on a vertically immersed surface as shown in figure,

$$P = w \cdot A \cdot \bar{x}$$



and the depth of centre of pressure from the liquid surface,

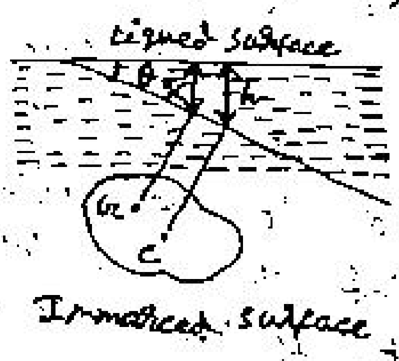
$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

Where, I_G = M.I of immersed surface about the horizontal axis through its c.g.

(3) Inclined immersed surface :

The total pressure on an inclined surface, as shown in figure,

$$P = w \cdot A \cdot \bar{x}$$



and the depth of centre of pressure from the liquid surface,

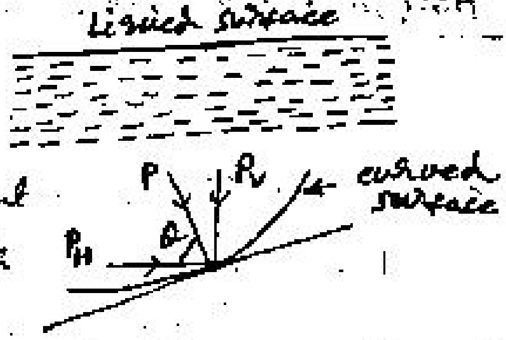
$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

Where, θ = Angle at which the immersed surface is inclined with the liquid surface.

(4) curved immersed surface:

The total force on the curved surface, as shown in figure,

$$P = \sqrt{(P_H)^2 + (P_V)^2}$$



and the direction of the resultant force on the curved surface with the horizontal is given by,

$$\tan\theta = \frac{P_V}{P_H} \text{ or, } \theta = \tan^{-1}\left[\frac{P_V}{P_H}\right]$$

where, P_H = Horizontal force on the curved surface and is equal to the total pressure on the projected area of the curved surface on the vertical plane, and

P_V = Vertical force on the curved surface and is equal to the weight of the liquid supported by the curved surface upto the liquid surface.

Archimede's Principle:

When a body is immersed wholly or partially in a liquid, it is lifted up by a force equal to the weight of liquid displaced by the body.

Concept of buoyancy:

The tendency of a liquid to uplift an immersed body, because of the uplift thrust of the liquid is known as buoyancy.

The force tending to lift up the body is called the force of buoyancy or buoyant force and is equal to the weight of the liquid displaced. The point through which the buoyant force is supposed to act, is known as centre of buoyancy.

It may be noted that,

(a) If the force of buoyancy is more than the weight of the liquid displaced, then the body will float.

(b) If the force of buoyancy is less than the weight of the liquid displaced, then the body will sink down.

Metacentre and metacentric height:

The metacentre may be defined as a point about which a floating body starts oscillating, when given a small angular displacement. It is denoted by M.

The metacentric height is the distance between the c.g of the floating body and the metacentre (M).

Mathematically, metacentric height,

$$GM = \frac{I}{V} - BG = BM - BG$$

Where, I = M.I of sectional area of the floating body at the water surface.

V = Volume of the body submerged in water, and

BG = Distance between the centre of buoyancy (B) and the c.g (G).

Concept of ~~buoyancy~~ Floatation:

(1) Stable equilibrium: If a body floating in a liquid return back to its original position, when given a small angular displacement, then the body is said to be in stable equilibrium.

(2) Unstable equilibrium: If a body floating in a liquid does not return back to its original position and heels further away when given a small angular displacement, then the body is said to be in unstable equilibrium.

(3) Neutral equilibrium: If a body floating in ... occupies a new position and remains at rest in this new position, when given a small angular displacement then the body is said to be in neutral equilibrium.

The condition of equilibrium for a floating and submerged body are as follows:

Sl No	Equilibrium condition	floating body	submerged body
1.	Stable	M lies above G	B lies above G
2.	Unstable	M lies below G	B lies below G,
3.	Neutral	M and G coincides	B and G coincides

~~Flow through~~

Fluid flow:

Types of fluid flow:

(1) Uniform flow: A flow, in which the liquid particles at all sections of a pipe or channel have the same velocities, is called a uniform flow.

(2) Non-uniform flow: A flow in which the liquid particles at different sections of a pipe or channel have different velocities, is called a non-uniform flow.

(3) Streamline flow: A flow, in which each liquid particles has a definite path and the paths of individual particles do not cross each other, is called a stream-line flow.

(4) Turbulent flow: A flow in which each liquid particle does not have a definite path and the paths of individual particles also cross each other, is called a turbulent flow.

(5) Steady flow: A flow in which the quantity of liquid flowing per second is constant, is called a steady flow. It may be uniform or non-uniform.

(6) Unsteady flow: A flow in which the quantity of liquid flowing per second is not constant, is called unsteady flow.

(7) Compressible flow: A flow, in which the volume of a fluid and its density changes during the flow, is called a compressible flow. All the gases are considered to have compressible flow.

(8) Incompressible flow: A flow, in which the volume of a liquid and its density does not change during the flow, is called an incompressible flow. All the liquids are considered as incompressible flow.

(9) Rotational flow: A flow in which the fluid particles also rotate (have some angular velocity) about their own axes while flowing, is called a rotational flow.

(10) Irrotational flow: A flow in which the fluid particles do not rotate about their own axes and retain their original orientation, is called an irrotational flow.

(11) One-dimensional flow: A flow, in which the streamlines of its moving particles are represented stream lines, is called an one-dimensional flow.

(12) Two-dimensional flow: A flow whose stream lines of its moving particles are represented by a curve, is called two-dimensional flow.

(13) Three-dimensional flow: A flow whose streamlines are represented in space, along the three mutually perpendicular directions, is called three-dimensional flow.

(II) Properties of fluid (Problems):

(01) Calculate the specific weight, density and specific gravity of one litre of liquid which weighs 7 N.

Solution: Volume 1 litre $= \frac{1}{1000} m^3 = 1000 \, cm^3$

Weight $= 7 \, N$

(i) Specific weight $(w) = \frac{\text{Weight}}{\text{Volume}} = \frac{7 N}{\left(\frac{1}{1000}\right) m^3}$

$= 7000 \, N/m^3$ Ans:

(ii) Density $(\rho) = \frac{w}{g} = \frac{7000}{9.81} = 713.5 \, kg/m^3$ Ans:

(iii) Specific gravity $= \frac{\text{Weight density of liquid}}{\text{Weight density of water}}$ or $\frac{\text{Density}}{} = \frac{713.5}{1000}$

$= 0.7135$ Ans: (Density of water $= 1000 \, kg/m^3$)

(02) Calculate the density, specific weight and weight of one litre of Petrol of specific gravity $= 0.7$.

Solution: Given: Volume 1 ltr $= 0.001 \, m^3$.

Specific gravity, $S = 0.7$

(i) Density $(\rho) = S \times 1000 \, kg/m^3 = 0.7 \times 1000 = 700 \, kg/m^3$ Ans:

(ii) Specific weight $(w) = \rho \times g = 700 \times 9.81$

$= 6867 \, N/m^3$ Ans:

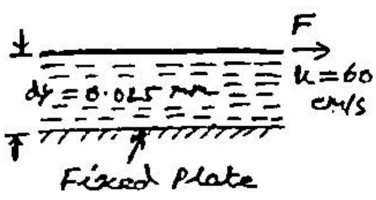
(iii) Weight (W) = Specific weight $\times$ Volume

$= 6867 \times 0.001 = 6.867 \, N$ Ans:

(03) A Plate 0.025 mm distant from a fixed Plate, moves at 60 cm/s and requires a force a force 2 N per unit area, $2 N/m^2$ to maintain this speed. Determine the fluid viscosity between the plates.

Solution: Given: Distance between Plates, $dy = 0.025 \, mm$
$= 0.025 \times 10^{-3} \, m$

Velocity of upper plate, $u = 60 \, cm/sec$
$= 0.6 \, m/sec$



Force on upper plate, $u = 2 \, N/m^2$

This is the value of shear stress τ.

Let the fluid viscosity between the plates is μ

We know, $\tau = \mu \cdot \frac{du}{dy}$

Where, du = change of velocity $= u - 0 = 0.60$ m/s

dy = change of distance $= 0.025 \times 10^{-3}$ m

$$\therefore \tau = 2 = \mu \times \frac{0.60}{0.025 \times 10^{-3}}$$

$$\therefore \mu = \frac{2 \times 0.025 \times 10^{-3}}{0.60} = 8.33 \times 10^{-5} \text{ Ns/m}^2$$

$$= 8.33 \times 10^{-5} \times 10 \text{ Poise} = 8.33 \times 10^{-4} \text{ Poise} \quad \underline{\text{Ans}}$$

[Unit of dynamic viscosity is N-S/m² (In MKS unit)

$\therefore$ 1 N-S/m² = 10 Poise

One Poise $= \frac{1}{10}$ N-S/m².]

(04) A flat plate of area 1.5×10^6 mm² is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15 mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity of 1 Poise.

Solution: Given: $\mu = 1$ Poise $= \frac{1}{10}$ NS/m²

Area of the plate, $A = 1.5 \times 10^6$ mm² $= 1.5$ m²

Speed of plate relative to another plate, $du = 0.4$ m/s

Distance $dy = 0.15$ mm $= 0.15 \times 10^{-3}$ m

We know, $\tau = \mu \frac{du}{dy}$

$$= \frac{1}{10} \times \frac{0.4}{0.15 \times 10^{-3}} = 266.66 \text{ N/m}^2$$

$\therefore$ (i) Shear force, $F = \tau \times \text{area} = 266.66 \times 1.5 = 400$ N $\underline{\text{Ans:}}$

(ii) Power required to move the plate at the speed 0.4 m/sec. $= F \times u$

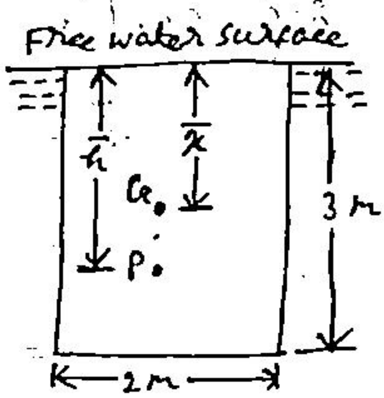
$$= 400 \times 0.4$$

$$= 160 \text{ W} \quad \underline{\text{Ans:}}$$

Hydrostatic (Problem):

(1) A rectangular plane surface is 2m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure of the plane surface when its upper edge is horizontal and (a) coincides with water surface, (b) 2.5 m below the free surface (water).

Solution: Given: Width of plane surface, $b = 2m$,
Depth of " " , $d = 3m$,



(a) Upper edge coincides with water surface:

∴ Total Pressure -

$$P = w \cdot A \cdot \bar{x} = \rho \cdot g \cdot A \cdot \bar{x}$$

where, $\rho = 1000 \ kg/m^3$, $g = 9.81 \ m/sec^2$

∴ $A = 3 \times 2 = 6 \ m^2$, $\bar{x} = \frac{1}{2}(3) = 1.5 \ m$

$$\therefore P = 1000 \times 9.81 \times 6 \times 1.5$$
$$= 88290 \ N \ \underline{\text{Ans}}$$

Depth of centre of pressure,

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

where, I_G = M.O.I about C.G of the area of surface,

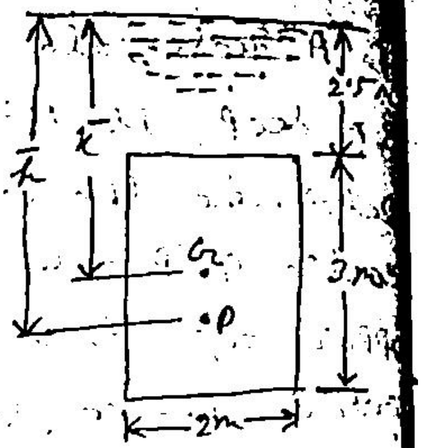
$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \ m^4$$

$$\therefore \bar{h} = \frac{4.5}{6 \times 1.5} + 1.5 = 2 \ m \ \underline{\text{Ans}}$$

(b) Upper edge is 2.5m below water surface.

$\therefore$ Total pressure,

$$P = \rho A g \bar{x}$$



where, $\bar{x}$ = Distance of c.g from free surface of water

$$= 2.5 + \frac{3}{2} = 4 \ m.$$

$$\therefore F = 1000 \times 9.81 \times 6 \times 4$$

$$= 235440 \ N \ \underline{\text{Ans:}}$$

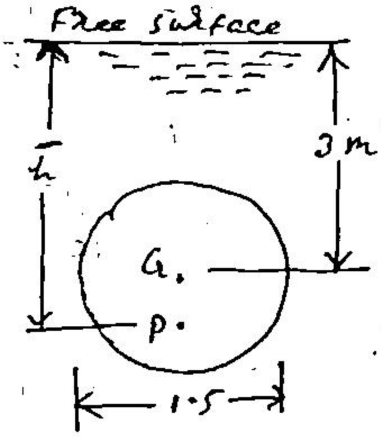
And centre of pressure,

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

$$= \frac{4.5}{6 \times 4} + 4$$

$$= 0.1875 + 4 = 4.1875 \ m \ \underline{\text{Ans:}}$$

(2) Determine the total pressure on a circular plate of dia 1.5m which is placed vertically in water in such a way that the centre of the plate is 3m below the free surface of water. Find the position of centre of pressure also.



<u>Solution:</u> Given; Dia of plate,

$$d = 1.5 \ m$$

$$\therefore \text{Area}, \ A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (1.5)^2$$

$$= 1.767 \ m^2$$

$$\bar{x} = 3 \ m$$

$\therefore$ Total pressure,

$$P = \rho \cdot g \cdot A \cdot \bar{x}$$

$$= 1000 \times 9.81 \times 1.767 \times 3$$

$$= 52002.81 \ N \ \underline{\text{Ans:}}$$

(III) Theory:

Manometer:

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as:

(a) Simple manometer

(b) Differential manometer

Mechanical gauge:

The mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight.
The commonly used mechanical pressure gauges are:

(a) Diaphragm pressure gauge.

(b) Bourdon tube pressure gauge.

(c) Dead weight " " .

(d) Bellow pressure gauge.

(III) Simple manometers:

A simple manometer consists of a glass tubes having one of its ends connected where pressure is is to be measured and other end remains open to atmosphere. common type of simple manometers are :

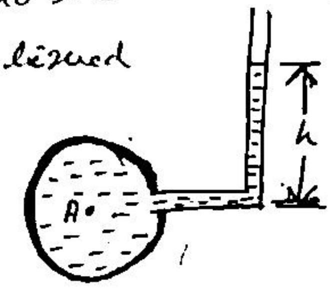
(1) Piezometer,

(2) U-tube manometer, and

(3) Single column manometer.

Piezometer :

It is simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in figure. The rise of liquid gives the pressure head at that point. If a point A, the height of liquid say water is h in piezometer tube, then pressure at A

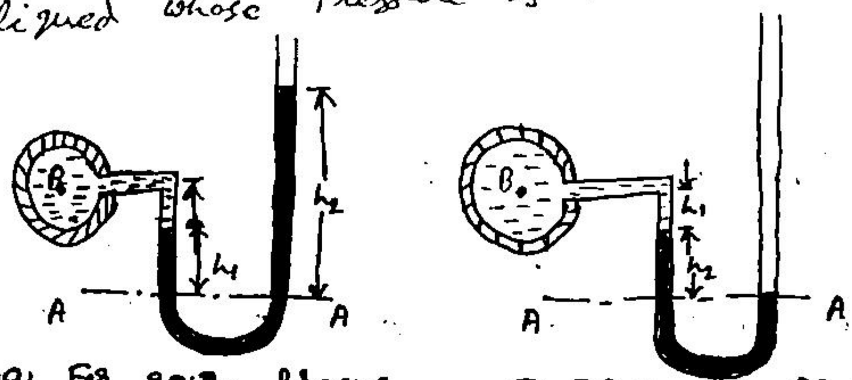


Piezometer

$$= \rho \times g \times h \quad N/m^2$$

U-tube manometer :

It consists of glass tubes bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in figure. The tube generally contains mercury or any other liquid whose specific gravity is greater than the sp. gravity of the liquid whose pressure is to be measured.



(a) For gauge pressure (b) For vacuum pressure

(a) For Gauge Pressure :

Let B is the point at which pressure is to be measured, whose value is P. The datum line is A-A.

Let, h_1 = Height of light liquid above the datum line.

h_2 = Height of heavy liquid above the datum line.

S_1 = Sp. gravity of liquid (light) = 0.9

S_2 = Sp. gravity of heavy liquid.

ρ_1 = Density of light liquid $= 1000 \times S_1$

ρ_2 = Density of heavy liquid $= 1000 \times S_2$

As the pressure is the same for horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= P + \rho_1 \times g \times h_1$

" " A-A in the right " $= \rho_2 \times g \times h_2$

Hence equating the two pressures $P + \rho_1 g h_1 = \rho_2 g h_2$

$$\therefore \quad P = (\rho_2 g h_2 - \rho_1 g h_1)$$

(b) For vacuum pressure :

For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in fig (b). Then

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + P$

Pressure head in the right column above A-A = 0

$$\rho_1 g h_1 + \rho_2 g h_2 + P = 0$$

$$P = -(\rho_1 g h_1 + \rho_2 g h_2)$$

(III) Single column manometer :

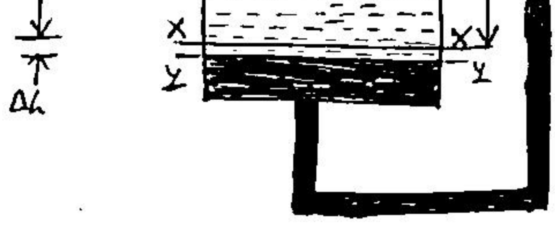
It is modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in fig below. Due to large cross sectional area of the reservior, ~~with be~~ for any variation of pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as :

(1) Vertical single column manometer.

(2) Inclined " " "

(1) # Vertical single column manometer :

The fig shows the vertical single column manometer. Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.



Vertical single column manometer

Let, Δh = Fall of heavy liquid in reservoir,

h_2 = Rise of heavy liquid in right limb.

h_1 = Height of centre of pipe above X-X.

P_A = Pressure at A, which is to be measured.

A = cross sectional area of the reservoir.

a = cross sectional area of the right limb.

S_1 = Sp. gravity of liquid in Pipe.

S_2 = Sp. gravity of heavy liquid in reservoir and right limb.

ρ_1 = Density of liquid in Pipe.

ρ_2 = Density of liquid in reservoir.

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in right limb.

$$\therefore A \times \Delta h = a \times h_2$$

$$\therefore \Delta h = \frac{a \times h_2}{A}$$

Now consider the datum line X-X as shown in figure.

Then pressure in the right limb above X-X

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb above X-X $= \rho_1 \times g \times (\Delta h + h_1) + P_A$

Equating this pressure we have,

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + P_A$$

or, $$P_A = \rho_2 g (\Delta h + h_2) - \rho_1 g (\Delta h + h_1)$$

$$= \Delta h (\rho_2 g - \rho_1 g) + h_2 \rho_2 g - h_1 \rho_1 g.$$

But from equation (i), $\Delta h = \frac{a \times h_2}{A}$

$$\therefore P_A = \frac{a \times h_2}{A} [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

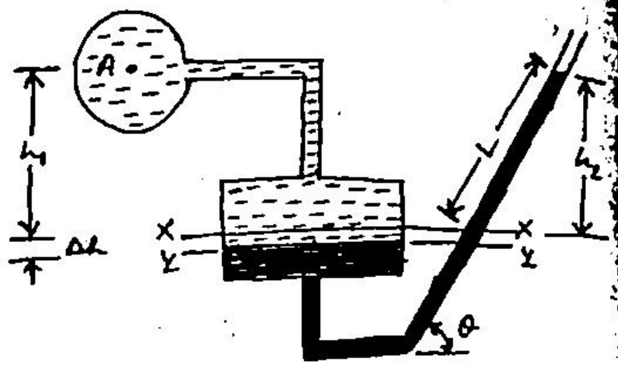
As the area A is very large as compared to a, hence ratio $\frac{a}{A}$ becomes very small and can be neglected.

Then, $P_A = h_2 \rho_2 g - h_1 \rho_1 g$

From this equation, it is clear that as h_1 is known and hence by knowing h_2 or rise of heavy liquid in the right limb, the pressure at A can be calculated.

(2) # Inclined single column manometer :

The figure shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liqued in the right limb will be more.



Inclined single column manometer

Let, L = Length of heavy liqued moved in right limb from x-x.

θ = Inclination of right limb with horizontal.

h_2 = Vertical rise of heavy liqued in right limb from x-x = $L \times \sin\theta$

From above equation, the pressure at A is,

$$P_A = h_2 \rho_2 g - h_1 \rho_1 g$$

Substituting the value of h_2, we get,

$$P_A = L\sin\theta \times \rho_2 g - h_1 \rho_1 g$$

(III) Differential manometers :

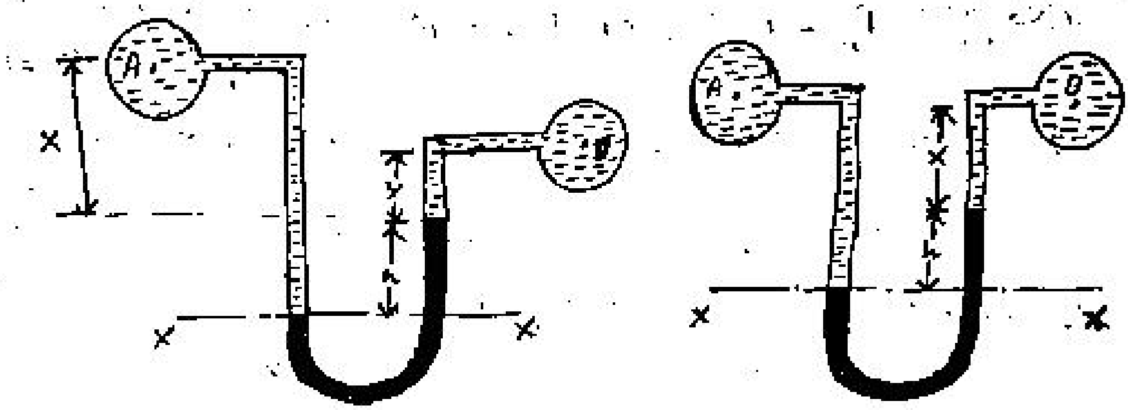
Differential manometers are the devices used for measuring the difference of pressure between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liqued, whose two ends are connected to the points, whose difference of pressure is to be measured. most commonly types of differential manometers are :

(1) U-tube differential manometer and

(2) Inverted U-tube differential manometer.

(I) U-tube differential manometer :

Below this figure shows the differential manometers of U-tube type.



(a) Two Pipes at different levels

(b) A and B are the same level

The figure (a) let the two points A and B are at different level and also contains liquids of different sp. gravity. These points are connected to the U-tube differential manometer. Let the pressure at A and B are P_A and P_B.

Let, h = Difference of mercury level in the U-tube.

y = Distance of the centre of B, from the mercury level in the right limb.

x = Distance of the centre of A, from the mercury level in the ~~right~~ left limb.

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X.

Pressure above X-X in the left limb $= \rho_1 g (h+x) + P_A$

Where P_A = Pressure at A

Pressure above X-X in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + P_B$

Where P_B = Pressure at B.

Equating the two pressure, we have

$$\rho_1 g (h+x) + P_A = \rho_g \times g \times h + \rho_2 \times g \times y + P_B$$

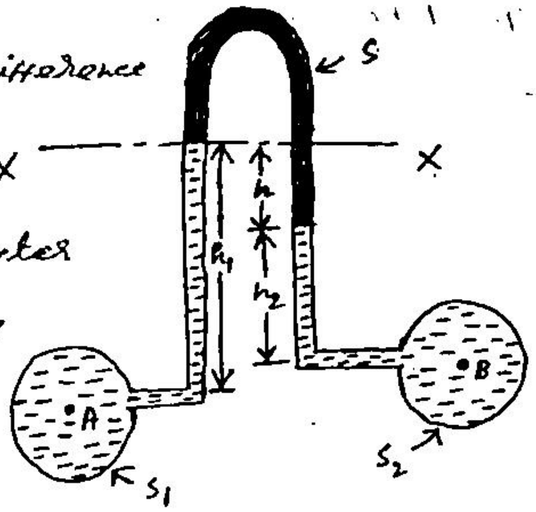
$$\therefore \quad P_A - P_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g (h+x)$$

$$\therefore\ P_A - P_B = g \times h\,(\rho_2 - \rho_1) + \rho_2 g y - \rho_1 g x$$

P.T.O. (Due)

(iii) Inverted U-tube Differential manometer:

It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the Paints whose difference of Pressure is to be measured.



It is used for measuring difference of low Pressures. This figure shows an inverted U-tube differential manometer connected to the two Paints A and B. Let the Pressure at A is more than the pressure at B.

Let, h_1 = Height of liquid in left limb below the datum line X-X.

h_2 = Height of liquid in right limb.

h = Difference of light liquid

ρ_1 = Density of liquid at A,

ρ_2 = Density of liquid at B.

ρ_s = Density of light liquid

P_A = Pressure at A.

P_B = Pressure at B.

Taking X-X at datum line. Then Pressure in the left limb below X-X $= P_A - \rho_1 \times g \times h_1$

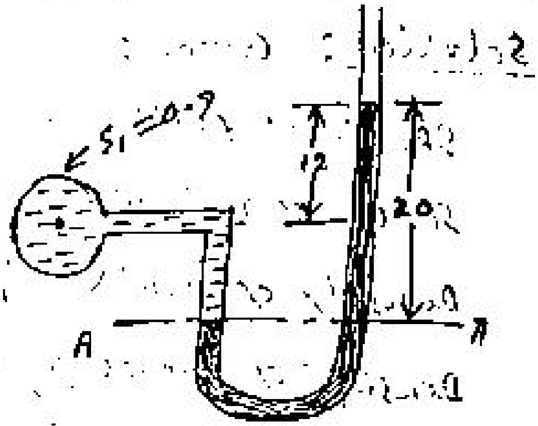
Pressure in the right limb below X-X

$$= P_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

(II) Manometer (Problem):

(1) The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid sp. gr 0.9 is flowing. The centre of the pipe is 12 cm above the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level is in the two limbs is 20 cm.

Solution: Given:



Sp. Gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000$

$= 0.9 \times 1000$

$= 900 \ kg/m^3$

Sp. Gr. of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = 13.6 \times 1000 \ kg/m^3$

Height of mercury level, $h_2 = 20 cm = 0.2 m$

Height of fluid from A-A, $h_1 = 20 - 12 = 8 cm = 0.08 m$

Let, p = Pressure of fluid in Pipe.

Equating the pressure above A-A, we get,

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

$$\Rightarrow p + 900 \times 9.81 \times 0.08 = 13600 \times 9.81 \times 0.2$$

$$\Rightarrow p = 26683 - 706 = 25977 \ N/m^2$$

$$= 2.597 \ N/cm^2 \ \text{Ans:}$$

(2) A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left ~~tube~~ from the centre of pipe is 15 cm below.

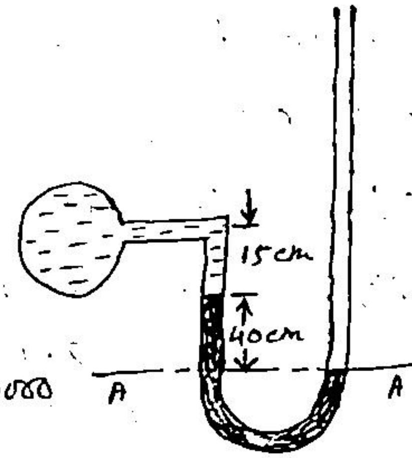
Solution: Given:

SP. Gr of fluid, $S_1 = 0.8$

SP. Gr of mercury, $S_2 = 13.6$

Density of fluid, $\rho_1 = 800$

Density of mercury, $\rho_2 = 13.6 \times 1000$



Height of mercury level, $h_2 = 40\,cm = 0.4\,m$

Height of liqued in left limb, $h_1 = 15\,cm = 0.15\,m$.

Let the pressure in pipe $= P$. Equating the pressure above datum line A-A, we get

$$\rho_2 g h_2 + \rho_1 g h_1 + P = 0$$

$$\Rightarrow \quad P = -(\rho_2 g h_2 + \rho_1 g h_1)$$

$$= -[13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$$

$$= -[53366.4 + 1177.2]$$

$$= -54543.6\ N/m^2$$

$$= -5.454\ N/cm^2 \quad \text{Ans:}$$

(3) A single column manometer is connected to a pipe containing a liqued of sp. gr 0.9 as shown in fig. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading shown in figure. The specific gravity of mercury is 13.6.

III: Fluid Flow :

Rate of flow or Discharge (Q) :

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid and compressible fluid the rate of flow or discharge is expressed as the volume and weight of fluid flowing across the section.

(i) For liquids the units of Q are m^3/s or litres/s

(ii) For gases " " " Q " kgf/s or Newtons/s

consider a liquid flowing through a pipe,

The discharge, $Q = A \times V$

where, A = cross sectional area of pipe,

V = Average velocity of fluid across the secⁿ.

continuity equation: (one dimensional flow)

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

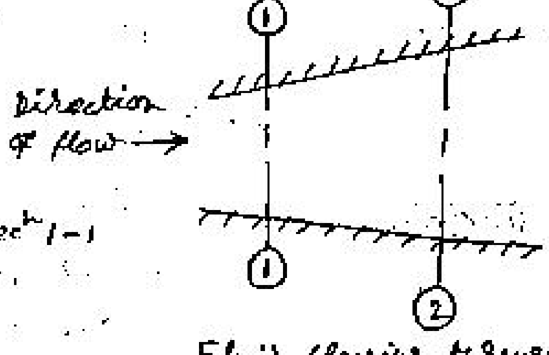
Proof :

consider two cross sections of a pipe as shown in fig.

Let, V_1 = Average velocity at cross-section 1-1

ρ_1 = Density at secⁿ 1-1

A_1 = Area of pipe at secⁿ 1-1



Fluid flowing through a pipe

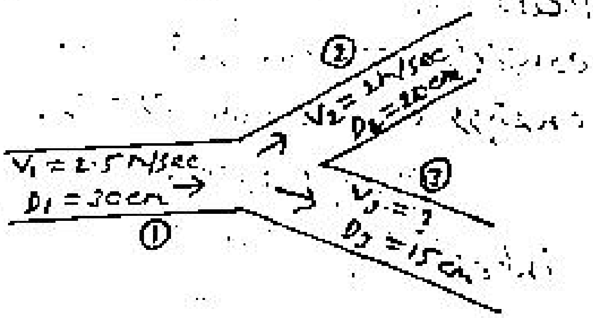
Also, we know that,

$$A_1V_1 = A_2V_2$$

(ii) $\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{0.007854}{0.01767} \times 5$

$= 2.22$ m/sec <u>Ans:</u>

Q2) A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity is 20 cm diameter pipe is 2 m/sec.



<u>Solution:</u> <u>Given:</u>

$D_1 = 30$ cm $= 0.30$ m

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.3)^2$

$= 0.07068$ m^2

$V_1 = 2.5$ m/sec

$D_2 = 20$ cm $= 0.2$ m

$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314$ m^2

$V_2 = 2$ m/sec

$D_3 = 15$ cm $= 0.15$ m

$A_3 = \frac{\pi}{4} D_3^2 = \frac{\pi}{4} \times (0.15)^2 = 0.01767$ m^2

Let, Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation,

$$Q_1 = Q_2 + Q_3 \quad \text{--- (i)}$$

(i) In pipe (1) the discharge, $Q_1 = A_1V_1$

$= 0.07068 \times 2.5$ m^3/sec

$= 0.1767$ m^3/sec <u>Ans:</u>

(ii) Value of V_3: $Q_2 = A_2V_2$

$= 0.0314 \times 2 = 0.0628$ m^3/sec

Substituting the values of Q_1 and Q_2 in eqn (i),

$$0.1767 = 0.0628 + Q_3$$

$$\therefore \quad Q_3 = 0.1767 - 0.0628 = 0.1139 \ m^3/s$$

But, $Q_3 = A_3 \times V_3$

or, $0.1139 = 0.01767 \times V_3$

or, $V_3 = 6.44$ m/sec Ans:

(iii) Bernoulli's theorem:

It states that in a steady ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic and potential energy or datum energy.

Proof: Bernoulli's equation is obtained by integrating the Euler's equation of motion.

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv$$

If flow is incompressible, ρ is constant and

$$\therefore \quad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\text{or,} \quad \frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\text{or,} \quad \frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

A Bernoulli's eqn in which

$\frac{p}{\rho g}$ = Pressure energy per unit weight of fluid or pressure head.

$\frac{v^2}{2g}$ = Kinetic head or kinetic energy per unit weight.

z = Potential energy per unit weight or Potential head